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14. ABSTRACT
This report presents a summary of the research supported by the AFOSR during the period June 1 2008 to May 31 2011 on the formulation and analysis of numerical methods for the analysis of failure in solids and structures of interest to the Air Force mission. Of special interest is the consideration of dynamic range, including dynamic fracture and failure. The development of new enhanced finite element methods that incorporate the discontinuous solutions characteristic of the failure of solids and structures has been one of the main outcomes of the project. The new methods include finite element formulations that address the special issues arising in the dynamic range, including for instance brittle/ductile failure mode transitions and crack branching. Similarly, the extension of these finite elements to the general three-dimensional range and the failure of partially saturated poroplastic media have also been addressed in this project.

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NUMERICAL ANALYSIS OF SOLIDS AT FAILURE

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Abstract

This report presents a summary of the research supported by the AFOSR during the period June 1 2008 to May 31 2011 on the formulation and analysis of numerical methods for the analysis of failure in solids and structures of interest to the Air Force mission. Of special interest is the consideration of dynamic range, including dynamic fracture and failure. The development of new enhanced finite element methods that incorporate the discontinuous solutions characteristic of the failure of solids and structures has been one of the main outcomes of the project. The new methods include finite element formulations that address the special issues arising in the dynamic range, including for instance brittle/ductile failure mode transitions and crack branching. Similarly, the extension of these finite elements to the general three-dimensional range and the failure of partially saturated poroplastic media have also been addressed in this project. Additional topics also considered during the later phases of the project with the goal of their consideration in full failure analyses include the formulation of invariant finite elements for thin Kirchhoff rods, and preliminary initial studies of growth in constitutive models of soft tissues.

KEYWORDS: finite elements with embedded strong discontinuities, material failure, dynamic fracture, crack branching, coupled poroplasticity, partially saturated media.

1. Objectives

The central goal of this research project has been the development of numerical methods for the analysis and numerical simulation of the failure of solids and structures of interest to the Air Force mission. The consideration of the dynamic range, including dynamic fracture and failure, has been one of the main focuses. A key goal is the development of new enhanced finite element methods that incorporate the discontinuous solutions characteristic of the failure of solids and structures, extending our previous efforts in this area. The discontinuities propagate independently of the underlying finite element mesh used in the approximation of the bulk response of the solid through the proper local finite element enhancements, thus leading to an efficient resolution of the failure and fracture of the solid. The completely local nature of these considerations is accomplished through a multi-scale treatment of the problem at hand, this being the main difference with other approaches that can be found in the current literature, avoiding in this way the prohibitive cost of these techniques based on a more global type enrichment as well as allowing an easy incorporation of the resulting methods in existing finite element codes. The key issue is the stable and accurate (locking free) numerical resolution of the kinematics associated with the discontinuities, in both the infinitesimal and finite deformation ranges, plane and three-dimensional settings.

The special characteristics of the dynamic problems being considered, involving for example brittle/ductile failure mode transitions and crack branching, require the formulation of the new methods. In this way, the new finite elements methods need to be able to accommodate crack branching, a common occurrence in the dynamic range and a situation not considered to date in the literature for finite elements with embedded strong discontinuities. In this setting, we have developed a new set of strategies to model crack branching including new finite elements with embedded branching, at the element level itself.

The generalization of the new finite element methods to the three-dimensional range has defined also a clear and challenging goal, arriving at new brick finite elements with embedded strong discontinuities in the infinitesimal range. Similarly, the numerical modeling of localized failures in partially saturated media showed also a clear challenge due to the coupled multi-physics nature of the problem, as it is also being addressed in the project. Furthermore, we have started an exploratory study for the extension of these ideas to the analysis of the failure of other structural/mechanical systems, including the finite element modeling of thin Kirchhoff rods and the constitutive modeling of growth in soft tissues in biomaterials. We can quote, in particular, the successful development of new invariant finite elements for the simulation of thin rods, in both the plane and three-dimensional setting with general underlying curved geometries, as they occur in the modeling of space tethers among other applications. The modeling of failure in these systems is an area of current and future development in our continuing work in the area of computational failure mechanics.

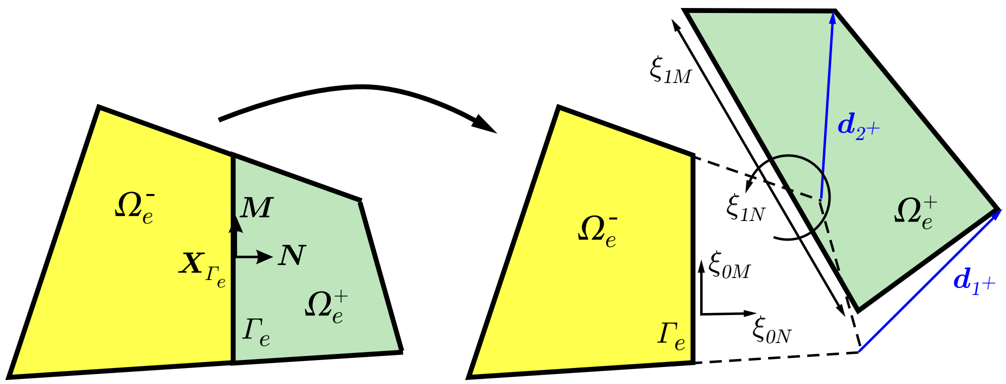


FIGURE 1 Plane finite elements with embedded discontinuities. The discrete strain field of the element is enhanced to include the strain associated to the separation mode shown here. It includes both constant and linear displacement jumps along the discontinuity, resulting in one part of element translating, rotating and stretching in the tangential direction with respect to the other part. These modes involve finite displacements in the finite deformation range, including a finite rotation in terms of the exponential map associated to the degree of freedom ξ_{1N}.

The improved stability properties of the newly developed integration algorithms are then supported by rigorous mathematical analyses of the discrete dynamical systems that they generate. We also have given a special attention to the actual implementation of the new algorithms in the context of the finite element method. The combination of all these results has led to powerful novel computational tools, with the sound theoretical basis necessary for the analysis of the complex practical problems of interest to the Air Force.

2. Brief Description of the Accomplishments

The major results accomplished under this project can be summarized as follows.

2.1. Plane finite elements with embedded strong discontinuities.

Our starting point is the new finite elements developed in the AFOSR project right preceding the current project and during the transition to the current project, in fact motivating it, successfully extending the formulation and application to dynamic problems. The elements incorporate a linear interpolation of the discontinuity jumps along the discontinuity. The key aspect is the proper enhancement of the strain field in the element. Both the infinitesimal and finite deformations ranges have been considered. The latter geometrical nonlinear case has required a careful consideration so the final formulation is frame indifferent and locking free. In this way, the elements can reproduce fully opened discontinuities (thus avoiding the so-called stress locking) and can accommodate general assumed strain strategies for the avoidance of e.g. volumetric locking in the bulk. Figure 1 represents the main idea behind the formulation of the new elements in the plane case: an element crossed by a discontinuity segment (assumed piece-wise straight in the considered representation) is enhanced with the strain field associated with the separation mode shown in the figure (two translations, one rigid finite rotation and one tangential uniform stretching of the separating part of the element) for linear discontinuity jumps along the discontinuity segment. We have fully developed the use of the new finite elements in the dynamic range, including challenging problems involving ductile/brittle failure mode transitions in ductile solids and dynamic fracture of brittle materials. We refer to [6,8,12]† and references therein for complete details of these developments

Figure 2 illustrates the performance of the new elements in the modeling of delamination of composite materials (DCB test). A rectangular specimen is pulled symmetrically on opposite sides at one end by imposing the separation displacement, measuring the required force; see Figure 2.a. The separation associated to the propagating delamination front is modeled as a discontinuity of the displacements in the whole solid. An exponential softening law relating the traction and the displacement jump is considered along the front, with an elastic orthotropic response for the bulk of the specimen. The individual elements crossed by the front are enhanced to resolve this separation. Figure 2.b show the computed deformation of the specimen at different times for two different mesh in the general nonlinear finite deformation range. We can note the stretching of those elements crossed by the strong discontinuity, leading to that large finite deformation (no magnification of the displacements is used in the figure). The computed reacting force vs. imposed conjugate displacement is shown in Figure 2.c for the fine mesh. We have included the results obtained with the finite deformation elements (right) but also the results with finite elements assumed infinitesimal kinematics of small strains (left). Interestingly the results for the latter match well the classical estimate obtained by considering Griffith theory of fracture with two separating cantilever beams modeling the fracturing specimen. This estimate, however, does not match the experimental results (as reported by Robinson & Song [1992], J. Composite Mat., 26, 1554-1577), whereas the simulation based on the finite elements developed in this work in the nonlinear deformation range do obtain well these experimental values. These results validate (and verify) well the finite elements in this work, besides indicating the need to consider large finite deformation kinematics in this type of problems, even if basic Mode I separation mode is involved.

This example involves quasi-static deformations, with the extension to the dynamic range having been one of the main goals of this project, as noted above. In particular, we have been able to capture successfully with the newly developed methods the failure mode transitions observed in the failure of notched steel specimens under the impact by a rigid projectile; see Figure 3.a. This transition is characterized by the full propagation through the specimen's width of a shear band originating from the tip of the notch for impact velocities higher than a certain critical value. For velocities below this value, the ductile

† The numbering of the references quoted correspond to the list of publications presented in Section 4 of this report, in page 17

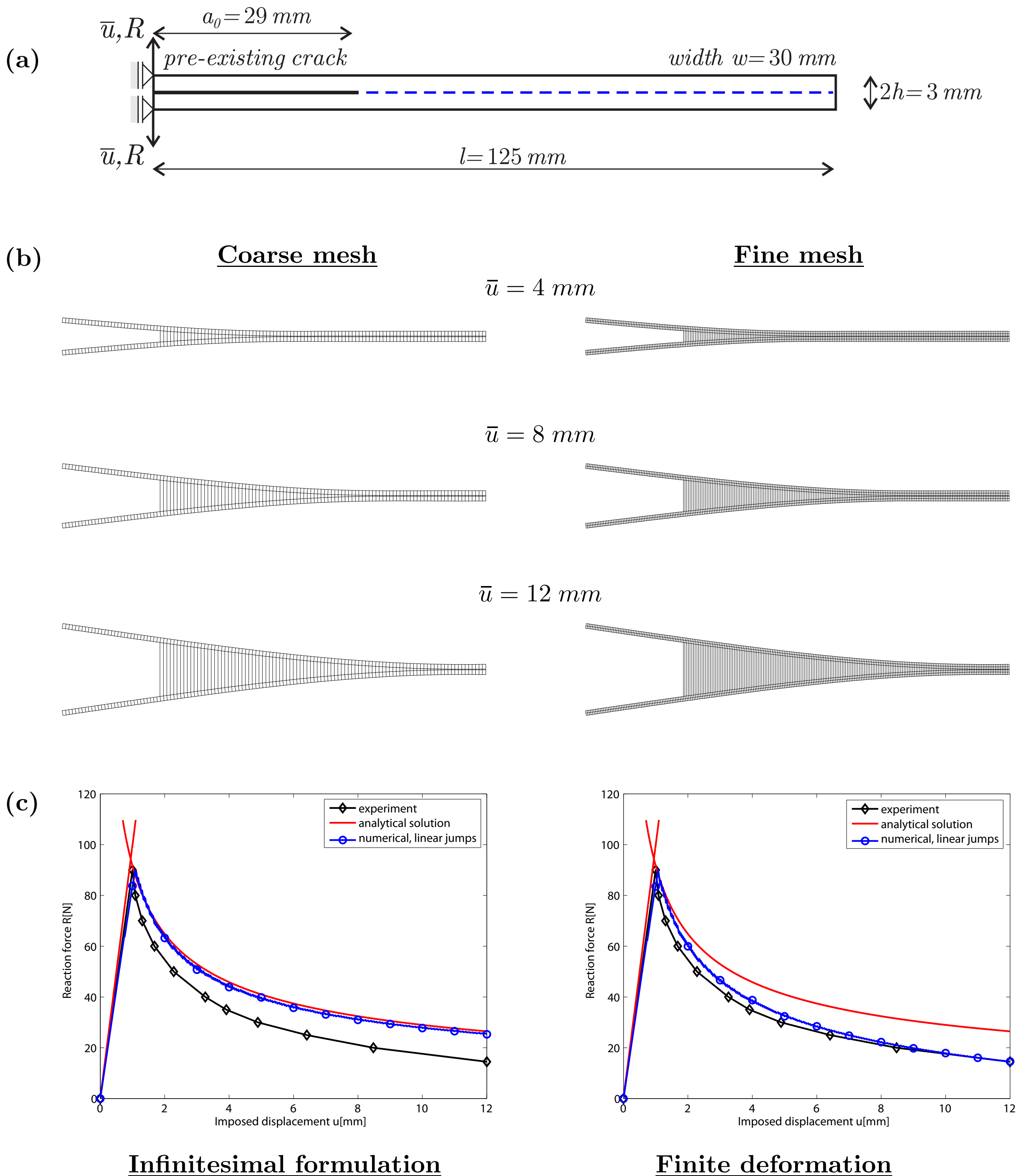
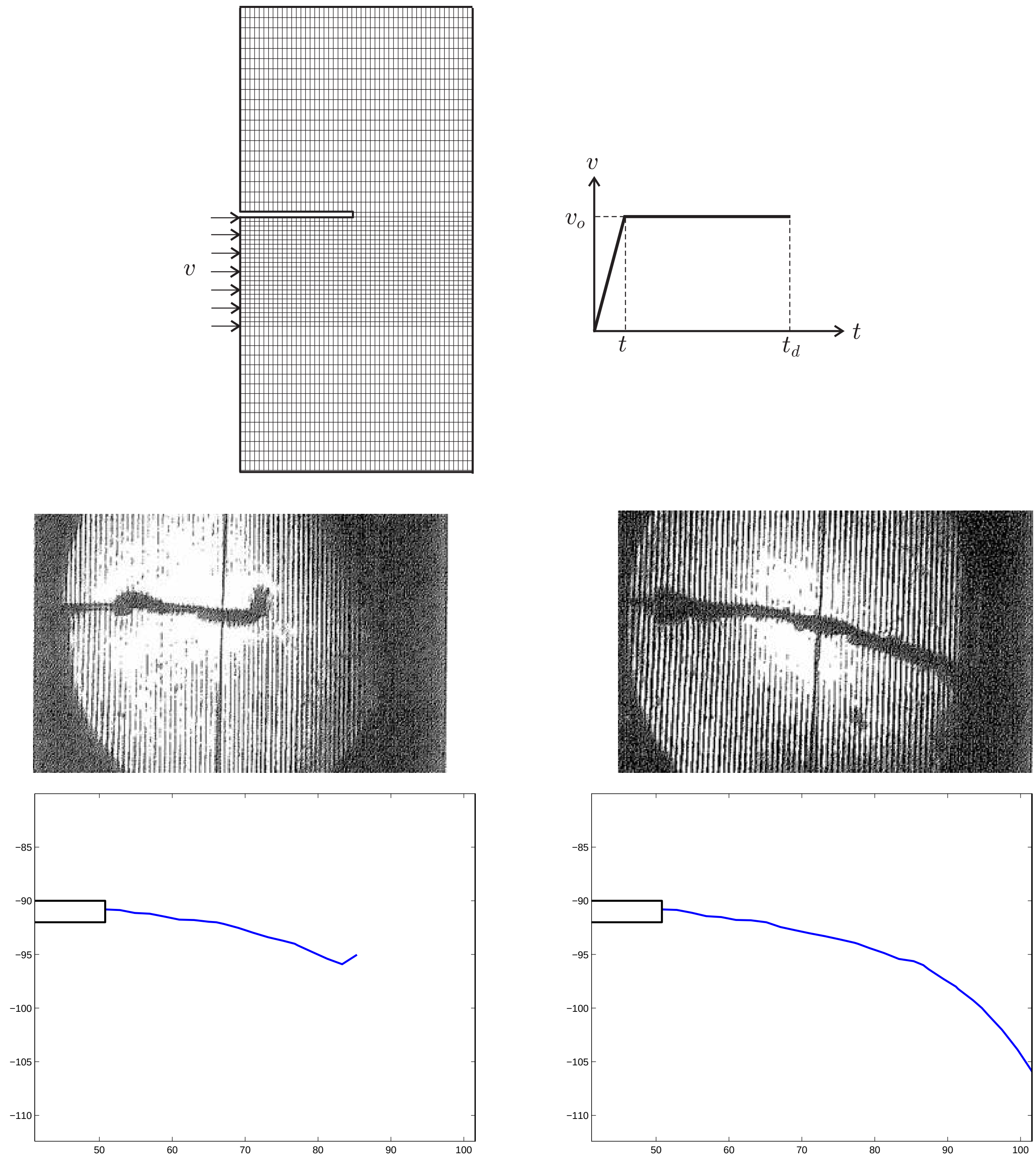


FIGURE 2 Double cantilever beam test for delamination in composites. **(a)** Problem definition: the specimen is pulled with transversal displacements $\overline{u}$, measuring the conjugate reaction R, as the delamination (discontinuity) front propagates. **(b)** Computed solutions with two different meshes (no magnifications). **(c)** Computed reaction versus imposed displacement using the infinitesimal elements (left) and the finite deformation elements (right) both with the fine mesh The infinitesimal solution matches well the basic estimate based on linear beam theory, whereas the finite deformation elements match well the actual experiments.

(a)

(b)



$\underline{v = 25\ m/s}$ $\underline{v = 30\ m/s}$

FIGURE 3 Failure mode transition in dynamic failures, **(a)** Problem definition. A notched specimen is impacted by a projectile below the notch, this being modeled by an imposed boundary velocity as shown. The material is assumed to follow finite strain J_2 plasticity. The considered finite element mesh is shown. **(b)** Solution for two different impact velocities, $v = 25\ m/s$ and $v = 30\ m/s$, below and above of the observed critical velocity for mode transition, from brittle to ductile failure. The experimental results reported by Zhou et al [1996] (J. Mech. Phys. Sol., 44, 981-1006) are shown, depicting the change of orientation of the propagating shear band for the lower velocity, compared with the full propagation of the shear band through the specimen for the higher one. The new finite elements with embedded discontinuities in the finite deformation range capture the same transition as shown in the bottom plots. Note that the discontinuity propagates through the mesh, independently of it, with the finite elements crossed by it enhanced accordingly.

slipping mode associated to the shear band changes in the middle of the specimen, developing a brittle crack at an angle. Physically, this transition can be traced to the tensile stress that appear due to waves propagating in the specimen. Remarkably, the new finite elements are able to accurately these complex fields, capturing very well not only the critical transition velocity but also the paths of the propagating shear band and brittle crack angle in the lower velocity. This example has allowed the full validation of the new finite elements, as well as the algorithm developed for the propagation of the discontinuities.

Figure 3 illustrates the performance of the new elements formulated this project for the finite deformation range in the modeling of mode transition in dynamic failure. A notched specimen is impacted by a projectile, leading to two different patterns of the propagating shear band for different velocities. See Figure 3.b for the experimental and computed results at two different impact velocities, below and above the critical value where the britle-ductile mode transition is observed. The bulk of the specimen follows an elastoplastic model (finite strain J_2 theory), with the shear band/crack being propagated based on the loss of ellipticity criterion of the resulting equations. The new elements resolve the shear band as a discontinuity through the newly developed enhancements, without the need of remeshing, following a linear softening cohesive law between tractions and displacement jumps. As observed in the experiments, the computed shear band spans the whole specimen for the large velocity, with the shear band stopping and deflecting as a brittle crack at an angle for the sub-critical velocity, matching again the response observed in the experiments. These results confirm the results obtained with an infinitesimal formulation, and the good performance of this approach in modeling this type of phenomena thanks to the improved numerical interpolations; we refer to [6,8] for details.

2.2. Basic finite element modeling of multiple strong discontinuities.

One of the main features in dynamic fracture is crack branching, with the associated multiple cracks propagating through the solid. This situation motivated us to develop algorithms for multiple strong discontinuities in the finite element context discussed above. In this way, we started the analysis of crack branching in dynamic fracture by extending the algorithm used for the propagation of the discontinuities in the static range to handle multiple discontinuities. In particular, for plane problems, we developed a propagation algorithm based on the connectivity graph of the underlying finite element mesh. In this setting, the discontinuities are defined by fronts propagating through this graph. The actual propagation through a given finite element is based on a physical condition in terms of the local state of stress and strain (the loss of ellipticity of the governing equations in general). After identifying the elements where this condition is reached, the propagation algorithm considers local geometric considerations. Due to this local character, the newly implemented version with multiple propagating fronts for the different discontinuities has proven to be also very efficient and reliable.

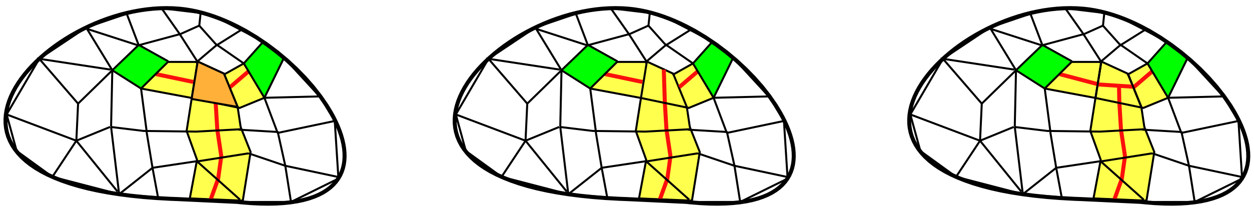


FIGURE 4 Different numerical strategies for modeling crack branching: element deletion, elements with single strong discontinuity, and finite elements with embedded branching.

Multiple attempts in the simulation of the actual branching of the discontinuities have indicated the adequacy of considering a criterion for the activation of several branches in a bifurcating discontinuity. In particular, based on the current physical understanding of the phenomenon of crack branching (still an open problem) we have considered the ratio of the tip velocity of the propagating discontinuities with the Rayleigh wave speed of the underlying matrix material. Computationally, this has required the development and implementation of an algorithm for the evaluation of the discontinuity's tip velocity. Multiple cases have been considered for the validation of this newly developed criterion with existing experimental tests and other reported numerical results. This validation and the refinement of the criterion continues.

In this context, we have considered several basic strategies for the numerical resolution of the actual branching (bifurcation) of the discontinuities. In particular, we have considered first the simple approach of opening additional discontinuity fronts in the elements contiguous to the element where branching has been detected based on the criterion described in the previous item. In the novel considered approach, new fronts start propagating through the mesh connectivity graph as indicated above, with the element where the branching was detected remaining with a single discontinuity. This strategy has proven to give better results (sharper resolution of the different branches) than the existing techniques in the literature based on simple element deletion; see Figure 4. We refer to [6,8] for further details.

Figure 5 illustrates some of the results obtained with this approach. A pre-cracked rectangular specimen is subjected to an imposed displacement in its upper and lower faces at different velocities v_o. The crack propagates first horizontally in a straight manner, eventually branching out of this plane. We have included plots of the obtained branched solutions and graphs showing the time evolution of the discontinuity tip velocity for two imposed velocities of $v_o = 3\ m/s$ and $15\ m/s$. When branching is not activated, the discontinuity is observed to reach the theoretical limiting value of the Rayleigh speed of the material (a ratio $r = v/c_r = 1$ in the plots). As observed experimentally, this pattern becomes unstable for lower values of this ratio and the crack branches out, decelerating

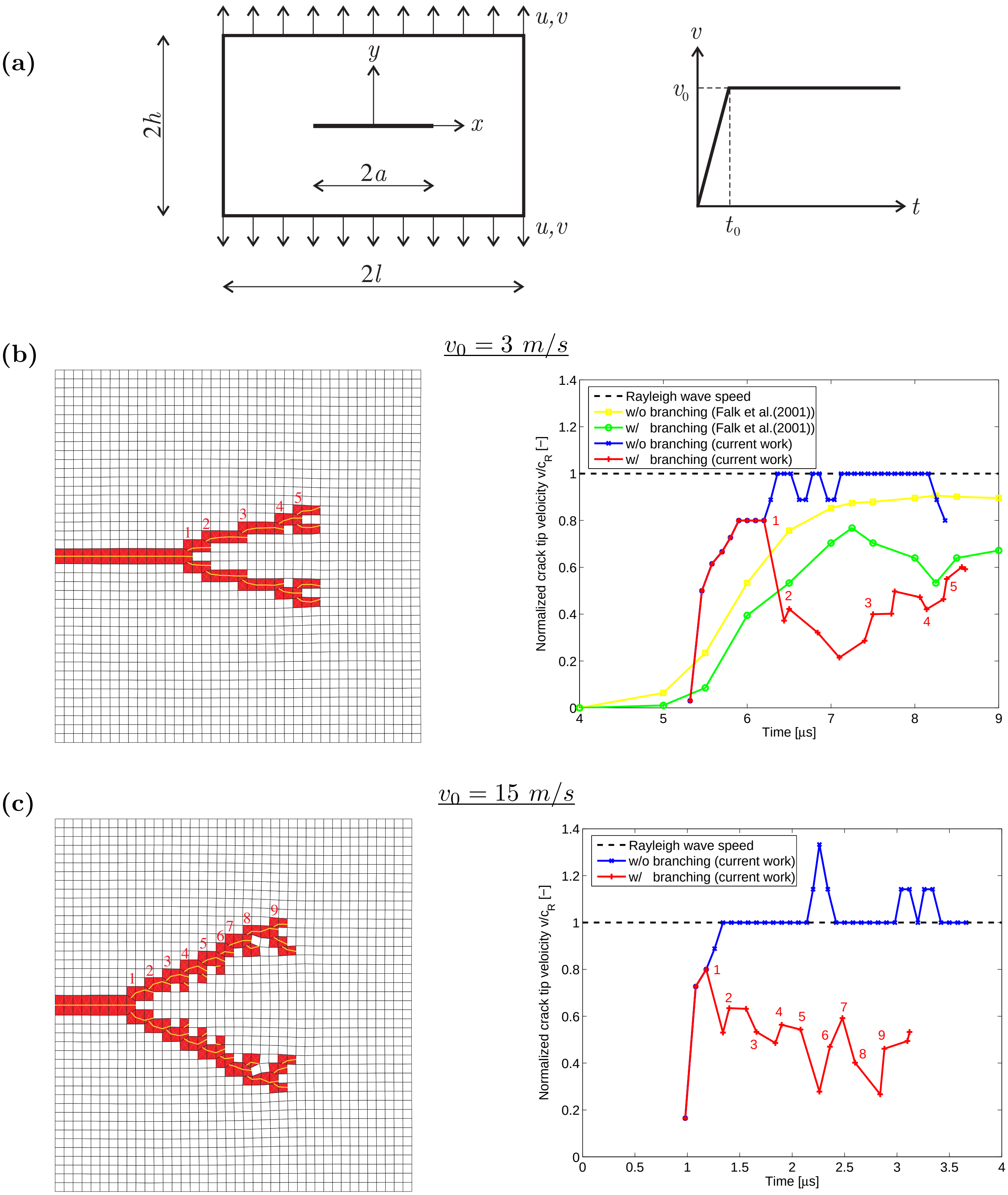


FIGURE 5 Crack branching. **(a)** Problem definition. Definition of the specimen's geometry ($l = 3\ mm$, $h = 1.5\ mm$, $a = 0.3\ mm$) and the variation of the imposed vertical velocity at the specimen's top and bottom faces, from 0 to different final velocities v_o at $t_o = 0.1\ \mu s$, remaining constant thereafter. **(b,c)** Computed paths (left column) and evolution in time of the tip velocity of the top crack (right column) for the imposed loading velocities $v_o = 3\ m/s$ and 15 m/s, repsectively.

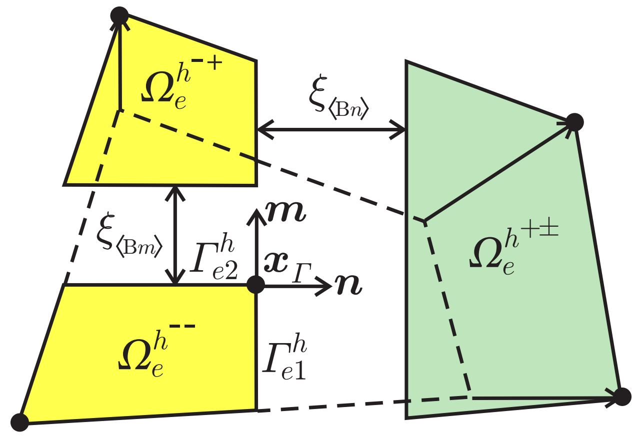


FIGURE 6 Finite elements with embedded branching. The discrete strain field of the underlying finite element is enhanced by incorporating the strain field associated to the branching mode, as shown in the figure

in the process. We observe how the newly developed numerical methods capture well this phenomenon. The plots include also some of the numerical results reported in the literature for comparison. Some of the newly formed branches develop fully while others stop, depending on the imposed top pulling velocity v_o. We can also observe the earlier activation of the branching and the steeper angle of the bifurcating branches for the higher velocity v_o, as observed in experiments.

2.3. Finite elements with embedded branching.

The aforementioned results in the numerical resolution of crack branching motivated us to develop finite elements that capture the actual branching in their interior. This feature has been accomplished by incorporating once more the associated strain modes to the discrete strain field of the element. We refer to the resulting elements as finite element with embedded branching. The results obtained with these elements indicate that they result in an improved resolution of the branching, leading in particular to richer bifurcating patterns as observed in experiments.

Figure 6 illustrates the separation modes considered in this case, with two discontinuities crossing in the element interior. A constant separation jump is considered for each of the branches in the normal direction, leading to a linear tangential displacement in one of the them (the vertical branch in the case illustrated in Figure 6) and the corresponding tangential stretching of the separating part of the element. This resulting strain mode, with different components in different parts of the element, is then introduced in the discrete strain field of the underlying element, following the ideas discussed above for a single discontinuity.

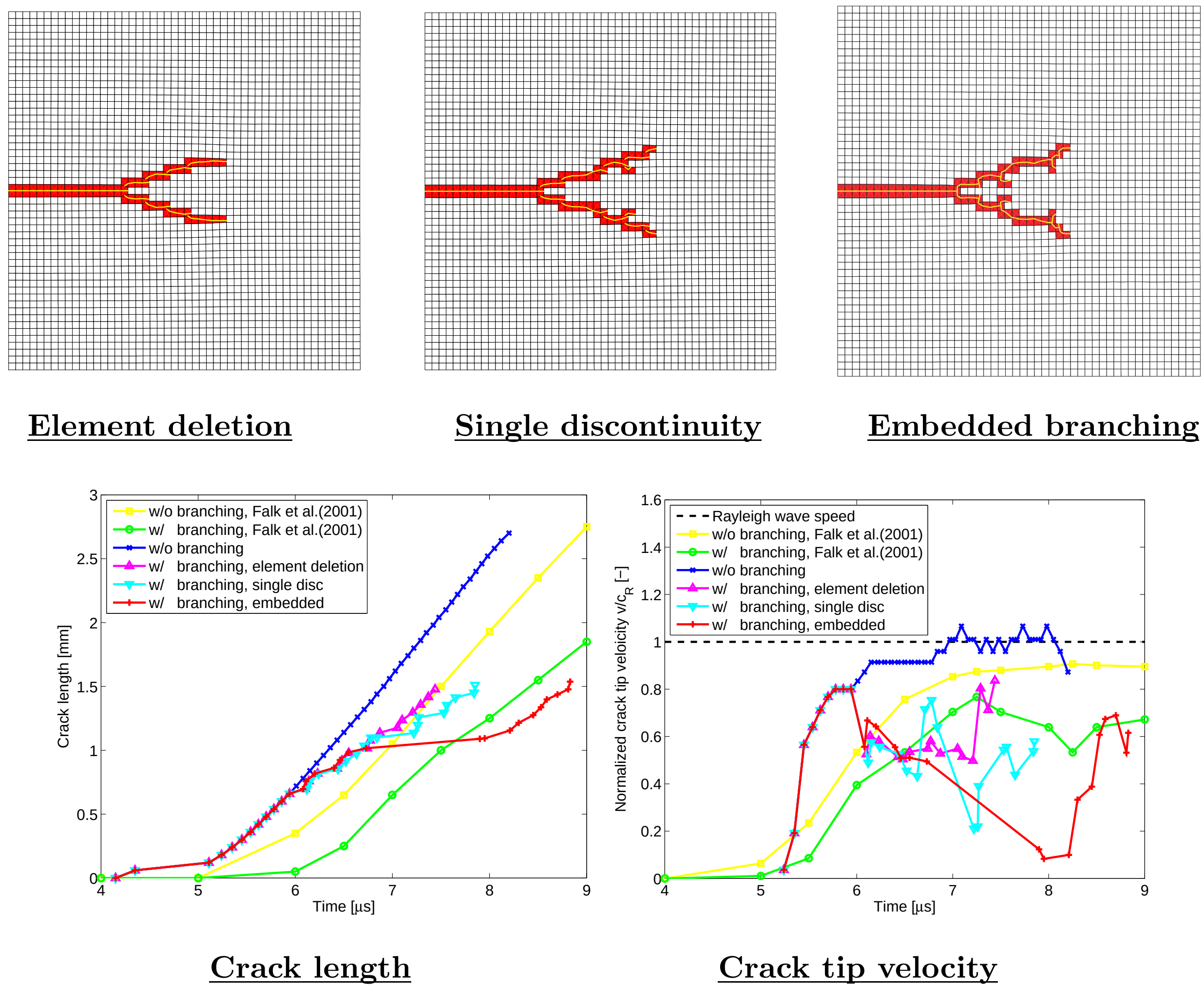
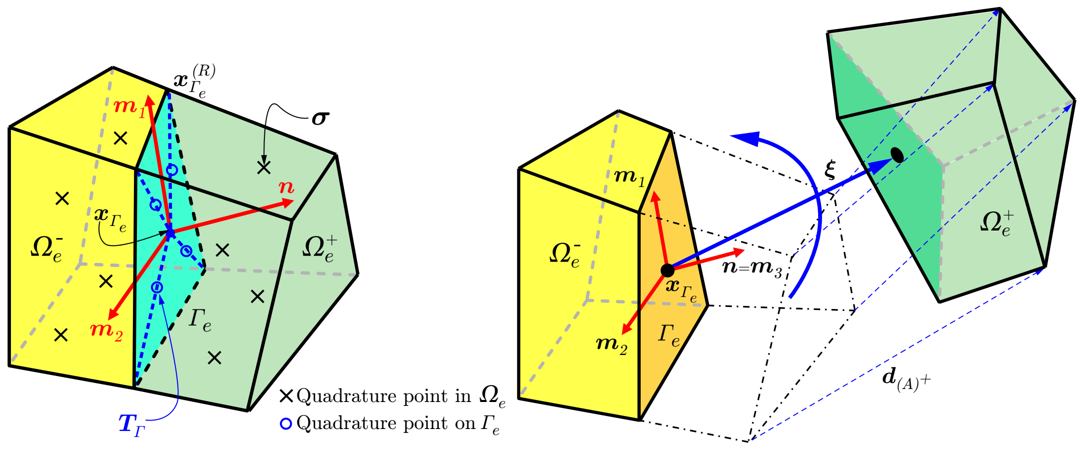


FIGURE 7 Finite element modeling of crack branching. Solutions obtained by the different strategies considered for the branched element for the problem defined in Figure 5.a and imposed top velocity of $v_o = 3m/s$: element deleting, element with single discontinuity (with the contiguous elements caching the bifurcating branches), and finite elements with embedded branching. Below, comparisons of the computed crack length and crack tip velocity, both measured on the main propagating branch, among these three different approaches and the results presented in the literature The plots include also the computed results not allowing branching, corresponding to the upper limit in crack length and tip velocity.

Figure 7 illustrates the results obtained with the resulting new elements for the same problem considered in Figure 5. We compare the results obtained by th three strategies sketched in Figure 4, namely, element deletion, single discontinuity with new contiguous fronts and finite elements with embedded branching. The evolution in time of the length and the crack tip velocity is shown for the main crack (top and bottom branches). We have included again the solution with no branching, with the crack reaching the Rayleigh speed of the material, basically maintaining a constant value afterward. The crack slows down, however, before reaching that value with the branching. Of the three strategies,



(a) **(b)**

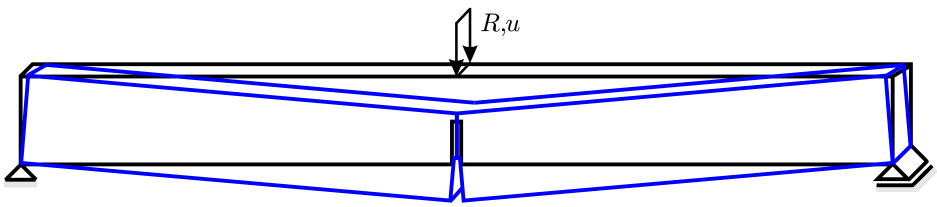
FIGURE 8 Three-dimensional finite elements with embedded discontinuities. **(a)** Finite element crossed by a segment of a discontinuity surface, with the associated local geometry including a surface quadrature rule by triangulation and a unique reference frame (note that the surface of discontinuity is not necessarily plane). **(b)** The discrete strain field of the element is enhanced to include the strain associated to the kinematics of the strong discontinuity, including both constant and linear displacement jumps along the discontinuity. This deformation results in one part of element translating, rotating and stretching/shearing in the tangential directions with respect to the other part.

we can observe a more remarked slow down with the new finite elements with embedded branching, being accompanied with the aforementioned richer pattern of branches (some active while others open and stop) as observed in experimental tests. Complete details of these developments have been presented in [7].

2.4. Three-dimensional elements with embedded strong discontinuities.

We have also started in this project the extension of the aforementioned finite elements to the three-dimensional case. We have first developed a new definition of the failure surfaces and the development of an algorithm for its propagation, in contrast to the curves representing these surfaces in the plane case. To this purpose, we have implemented a new locally-based (i.e. at the element level) algorithm that defines the general surfaces in the three-dimensional space as a level set of the solution of a heat-type problem determined by the detected normal to the failure surface, in contrast with a similar tracking algorithm but involving the global solution of such a problem as it has been proposed in the literature. We have also developed new brick finite elements that incorporate the proper separation modes for capturing the cracks and other localized failures of interest. The modes include constant and linear interpolations of the displacement jumps between the two separating parts of the element; see Figure 8 for an illustration of these modes.

Figure 9 illustrates these developments, showing the results obtained for the benchmark



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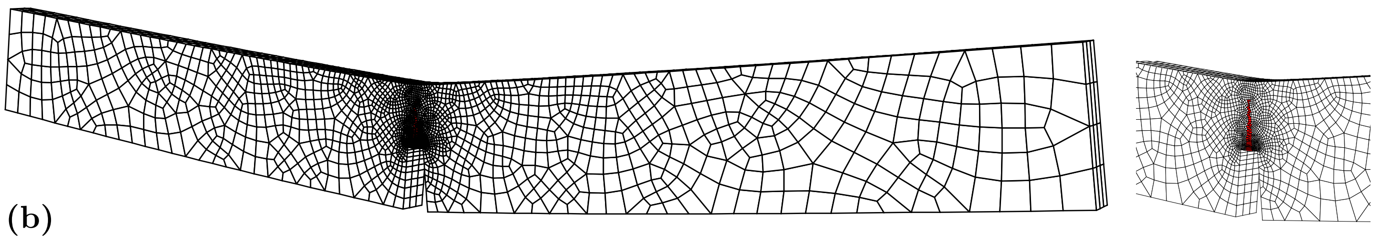

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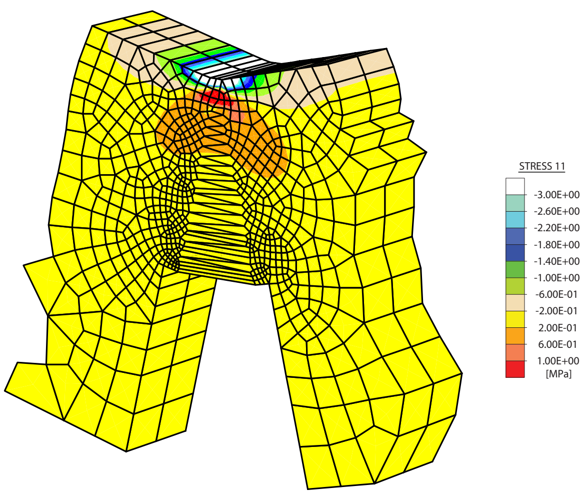

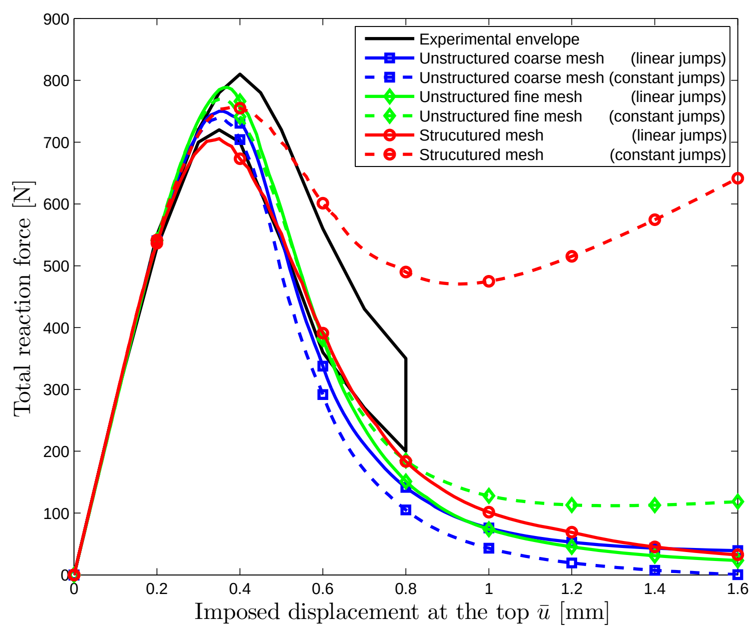

(d) **(e)**

FIGURE 9 Three-point bending test. **(a)** A notched beam simply supported at both ends is loaded at the top center by an imposed displacement u down (measuring the corresponding reaction force R), considering a linear elastic response up to the tensile strength of the material (concrete) followed by a power law cohesive softening law. **(b)** Computed deformation at $u = 0.50$ mm with zoom depicting the elements with an active discontinuity segment shown in red (fine unstructured mesh of 6,048 brick elements). **(c)** Stress σ_{11} (normal stress in the direction of the beam axis) at $u = 1.20$ mm showing the release of the stress due to the presence of the crack, with a small tensile head still opening it (coarse unstructured mesh of 1,632 brick elements). **(d)** Reaction versus imposed displacement laws obtained for different elements with different meshes in comparison with the experimental envelope. The locking of elements based on constant approximations of the displacement jumps can be observed by the unphysical hardening response, avoiding the full release of the stresses in the crack, in contrast with the performance of the newly proposed elements with embedded discontinuities with linear jump interpolations.

problem of the three-point bending test of a concrete beam. As depicted in Figure 9.a, a notched beam is loaded by an imposed transversal displacement at the top center while simply supported at both ends. The material model is linear elastic up to the concrete's tensile strength followed a localized softening cohesive law along the propagating crack. A power law with damage is considered. Figure 9.b shows the deformed configuration computed with a mesh of $6,048$ brick elements (fine unstructured mesh). The zoom in this figure shows details of the elements with active enhancement capturing the propagating crack. Note that the path of the propagating crack is not predetermined in the mesh. Figures 9.c depicts the distribution of the stress σ_{11} (normal stress along the direction of the axis of the beam) for a mesh with $1,632$ brick elements (coarse unstructured mesh), showing the tensile head that propagates the crack and the release of the stresses by the discontinuity. This release results in the softening response of the beam as shown in Figure 9.d depicting the computed reacting force with the imposed displacement. We can observe the stress locking obtained with elements based on a piece-wise constant interpolation of the displacement jumps (dashed lines), leading even to a hardening response not allowing the full release of the stresses along the crack. This situation is to be contrasted with the solutions computed with the new brick finite elements with linear jumps, obtaining the expected full release of those stresses and matching well also the experimental results.

Preliminary results along these lines have been presented in [4]. We are currently preparing a full set of publications presenting these results. Current and future work also include the consideration of the finite deformation range, including the consideration of finite strain plasticity as discussed above for plane problems.

2.5. Strong discontinuities in partially saturated media at failure.

We have characterized strong discontinuities in partially saturated media to model the localized failure of these systems, like dilatant shear layers. The main challenge has been the consideration of the mass flows of the different fluid phases in the porous solid, typically water and gas. In this setting, the strong discontinuities involve not only discontinuous displacements but also discontinuous fluids flow vector fields, leading to singular distributions of the fluid contents (fluid accumulation and drainage along the failure surface) while the fluid pressures and saturation degrees remain continuous. The main idea that has allowed the success of the proposed approach has been the multi-scale treatment of the problem, capturing in particular the kinematics of the strong discontinuities and the resulting local fluid flows in the small scales. These considerations apply not only to the theoretical formulation but also to the development of new finite elements incorporating the singular fields through the proper enhancement of the discrete strain and flow vectors. The case of partially saturated poro-plasticity in the infinitesimal range has been considered to date.

Figure 10 illustrates these particular developments, considering the failure associated to the excavation of a shallow circular tunnel. Figure 10.a defines the considered problem,

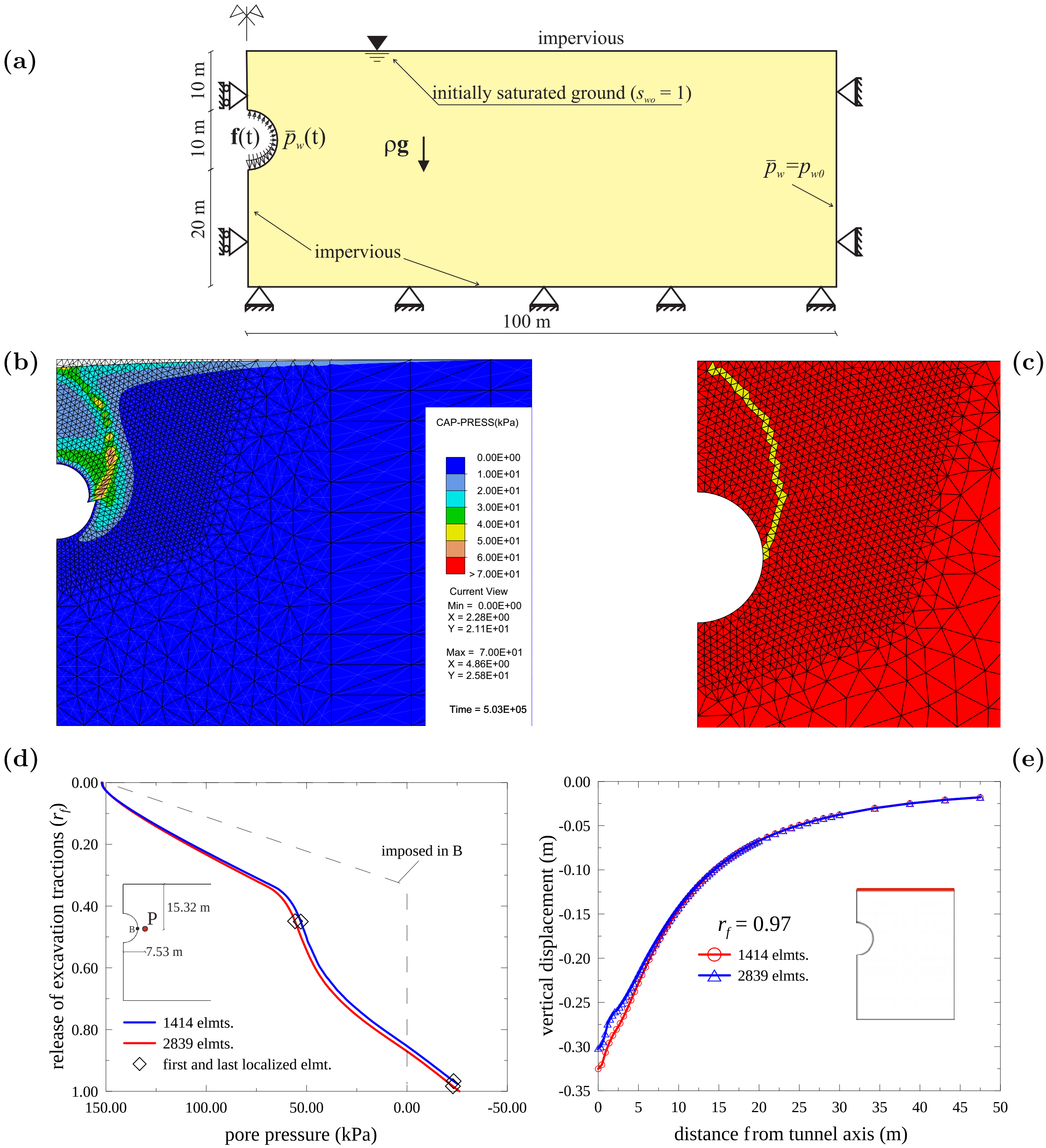

FIGURE 10 Excavation of a shallow circular tunnel. **(a)** Geometry and boundary conditions; the tractions and pore pressure are linearly released around the tunnel cavity. **(b)** Computed distribution of the capillary pressure ($p_c = p_{water} - p_{gas}$) in the partially saturated medium at failure. We note the concentration along the failure surface in the right top side of the tunnel, matching experimental observations. **(c)** Details of the mesh around the tunnel showing the finite elements with an active discontinuity enhancement modeling the failure surface. **(d)** Evolution of the pore pressure at a point close to the tunnel as the tractions along the tunnel surface are released (r_f decreasing from 0.0 to 1.0). **(e)** Distribution of the vertical displacement (subsidence) of the ground surface. The solution for two different meshes (1,414 and 2,839 elements) are depicted in these last two plots. The good agreement for the two meshes shows in particular the mesh-size independence (objectivity) of the developed numerical approach.

showing the geometry and boundary conditions considered in the problem. It involves an initially fully saturated medium around the tunnel under plane strain conditions, with half the domain considered due to the symmetry in the problem. The loading is defined by the gradual release of the mechanical tractions and pore pressures around the tunnel cavity, simulating the effects of the excavation. Figures 10.b and 10.c show some details of the computed solution. In particular, we can see the distribution of the capillary pressure (difference between the water and gas pressures) at failure. The concentration along the computed failure surface is to be noted. This surface can be more clearly seen in Figure 10.c., where we have highlighted the elements with active enhanced modes modeling the strong discontinuity in a detail plot of the finite element mesh around the tunnel. The computed solution agrees well with the observed patterns in actual field and experimental measurements. Figures 10.d and 10.e compare the solutions obtained with two different meshes, the fine mesh shown in Figure 10.b with $2,838$ finite elements and a coarser mesh of $1,414$ elements. They depict the evolution of the pore pressure at the shown point close to the tunnel and the distribution of the vertical displacement at the open ground surface (subsidence). The good agreement between the two meshes shows in particular the mesh-size independence (also known as objectivity) of the proposed formulation to capture these localized failures, that is, the correct resolution of the localized failure mechanisms associated to the failure surface. We refer to [3,9] for complete details.

2.6. Other developments.

We have also developed new finite elements for the modeling of thin Kirchhoff rods. The focus so far has been in the formulation of a new invariant interpolation of the rod's generalized displacements based on Hermitian shape functions given the C^1 continuity requirements of the kinematics of such a rod. The invariance of the newly proposed formulation is to be contrasted with approaches available in the literature for curved elements not preserving the rigid-body modes of the underlying physical theory (and hence leading to finite elements not satisfying the basic equilibrium relations between nodal forces and moments). We have presented these developments in [1,2]. The infinitesimal range has been considered so far, including the torsion of the rod in the three-dimensional range. Extensions currently under development include the nonlinear finite deformation range and the considerations of rods at failure through softening hinges in the framework of strong discontinuities.

Other explored areas with direct results in submitted papers include the formulation of a new constitutive model for growth in soft tissues, in a treatment based on a multiplicative decomposition similar to the inelastic models for failure considered in this project. These results have been presented in [5,13,14]. Besides, the papers [10,11] on energy-momentum numerical integration in nonlinear dynamics of elastoplastic solids were prepared during this project, based on our previous work with AFOSR.

3. Publications under AFOSR Support

The following papers have appeared and/or have been prepared during the performance of this grant acknowledging AFOSR support (2008/2011):

1. Armero, F. and Valverde, J. [2011] "Invariant Hermitian Finite Elements for Thin Kirchhoff Rods. I: The Linear Plane Case," *Computer Methods in Applied Mechanics and Engineering,* in press, available online, doi:10.1016/j.cma.2011.05.009.

2. Armero, F. and Valverde, J. [2011] "Invariant Hermitian Finite Elements for Thin Kirchhoff Rods. II: The Linear Three-Dimensional Case," *Computer Methods in Applied Mechanics and Engineering,* in press, available online, doi:10.1016/j.cma.2011.05.014.

3. Callari, C.; Armero, F. and Abati, A. [2010] "Strong Discontinuities in Partially Saturated Poroplastic Solids," *Computer Methods in Applied Mechanics and Engineering,* 199, 1513-1535.

4. Armero, F. and Kim, J. [2010] "Progress in the Formulation of Finite Elements with Embedded Discontinuities to Model Failure in Solids," Proceedings of the 10^{th} International Conference on Computational Structures Technology (CST2010), ed by B.Topping et al, Valencia, Spain, September 14-17, 2010.

5. Oller, S.; Bellomo, F.J.; Armero, F. and Nllim G. [2010] "A Biological Growth Model for Soft Tissue Under Stress Driven and Biological Availability," Proceedings of 9^{th} World Congress on Computational Mechanics (WCCM 9), Sydney, Australia, July 19-23 2010.

6. Armero, F. and Linder, C. [2009] "Numerical Simulation of Dynamic Fracture Using Finite Elements with Embedded Discontinuities," *International Journal of Fracture,* 160, 119-141.

7. Linder, C. and Armero, F. [2009] "Finite Elements with Embedded Branching," *Finite Elements in Analysis and Design,* 45, 280-293.

8. Armero, F. and Linder, C. [2009] "Numerical Modeling of Dynamic Fracture," Proceedings of ECCOMAS Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN09), Rhodes, Greece.

9. Callari, C.; Armero, F. and Abati, A. [2009] "Strong Discontinuities in Partially Saturated Poroplastic Solids," Report no. UCB/SEMM-2009/02, University of California at Berkeley. (revised version appeared in *Computer Methods in Applied Mechanics and Engineering,* see list above).

10. Armero, F. [2008] "Energy-Momentum Algorithms for the Nonlinear Dynamics of Elastoplastic Solids," refereed article in Theoretical, Modeling and Computational

Aspects of Inelastic Media, ed. by B.D. Reddy, IUTAM Book Series, Springer (published October 1, 2008).

11. Armero, F. [2008] "Energy-Momentum Algorithms for Nonlinear Solid Dynamics and their Assumed Strain Finite Element Formulation," refereed article in Computational Structural Dynamics and Earthquake Engineering, ed. M. Papadrakakis et al, CRC Press/Balkema, Taylor & Francis, Leiden (published December 9, 2008).
12. Linder, C. and Armero, F. [2008] "Modeling Crack Branching Using Finite Elements with Strong Discontinuities," Report no. UCB/SEMM-2008/02, University of California at Berkeley
13. Bellomo, F.J.; Oller, S.; Armero, F. and Nallim, G. [2011] "A General Constitutive Model for Vascular Tissue Considering Stress Driven Growth and Biological Availability," submitted.
14. Bellomo, F.J.; Armero, F.; Oller, S. and Nallim, G. [2011] "A Constitutive Model for Tissue Adaptation: Necrosis and Stress Driven Growth," submitted.

4. Interactions, Conference Contributions

The results obtained in this research project have been presented in the following conferences/seminars during the performance of this grant (2005/2008):

1. "Energy-Momentum Algorithms for Nonlinear Coupled Thermo-Elastodyna- mics," invited keynote lecture, 8^{th} World Congress on Computational Mechanics (WCCM 8), Venice, Italy, June 30-July 4 2008.
2. "Modeling of Dynamic Fracture using Finite Elements with Embedded Strong Discontinuities," invited contribution, 8^{th} World Congress on Computational Mechanics (WCCM 8), Venice, Italy, June 30-July 4 2008.
3. "Analysis of Strong Discontinuities in Partially Saturated Poroplastic Solids," invited contribution, 8^{th} World Congress on Computational Mechanics (WCCM 8), Venice, Italy, June 30-July 4 2008.
4. "Progress in the Formulation of Energy-Momentum Schemes for Nonlinear Solid and Structural Dynamics," invited seminar, School of Civil Engineering, Universitat Politècnica de Catalunya, Barcelona, Spain, July 21 2008
5. "Energy-Momentum Algorithms for Finite Strain Plasticity," invited seminar, Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, September 29 2008.

6. "Finite Elements with Embedded Strong Discontinuities and Branching for the Modeling of Failure in Solids," plenary lecture, ENIEF08, Congress of the Argentinian Association of Computational Mechanics, San Luis, Argentina, November 10-13, 2008.

7. "Energy-Momentum Algorithms for the Dynamics of Nonlinear Solids and Structures," invited seminar, Department of Civil and Environmental Engineering, Facultad de Ingeniería, Universidad Nacional de Cuyo, Mendoza, Argentina, November 14 2008.

8. Series of invited seminars in four different universities in Spain, March 23-27 2009. "Energy-Momentum Algorithms for the Nonlinear Dynamics of Elastoplastic Solids," given at the U.of Granada (CEE Dept.) and U. of Seville (ME Dept). "Finite Elements with Embedded Strong Discontinuities and Branching for the Modeling of Failure in Solids," given at the Politecnical U. of Madrid (ME Dept.) and U. of Zaragoza (ME Dept).

9. "Numerical Modeling of Dynamic Fracture," invited keynote lecture, 2^{nd} International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN09), Rhodes, Greece, June 22-24 2009.

10. "Finite Elements for Kirchhoff Rods," Congress on Numerical Methods in Engineering (METNUM 2009), SEMNI/APMTAC, Barcelona, Spain, June 22-July 2 2009.

11. "Finite Elements with Embedded Strong Discontinuities and Branching for Modeling Failure," plenary lecture, X International Conference on Computational Plasticity (COMPLAS 10), Barcelona, Spain, September 2-4 2010.

12. "Finite Elements with Embedded Strong Discontinuities for the Modeling of Failure in Solids," invited seminar, Department of Materials Science and Engineering, University of North Texas, Denton TX, October 2 2009.

13. "Finite Elements with Embedded Strong Discontinuities for the Modeling of Failure in Solids," plenary lecture, GTO10, Congress of the Mexican Association of Computational Mechanics, Guanajuato, Mexico, February 2-5, 2010.

14. "Finite Elements with Embedded Strong Discontinuities for the Modeling of Failure in Solids," invited seminar, Department of Civil and Environmental Engineering, Vanderbilt University, Nashville TN, February 15 2010.

15. "Finite Elements for Kirchhoff Rods," IV European Conference on Computational Mechanics (ECCM 2010), ECCOMAS, Paris, France, May 16-21 2010.

16. "Strong Discontinuties in Partially Saturated Media at Failure," invited contribution, 16^{th} US National Congress of Theoretical and Applied Mechanics (USNCTAM 2010), Penn State, State College PA, June 27-July 2 2010.

17. “Advances in Finite Elements with Embedded Discontinuities for the Modeling of Material Failure,” invited contribution, 9^{th} World Congress on Computational Mechanics (WCCM 9), Sydney, Australia, July 19-23 2010.

18. “An Analysis of Strain Localization in a Porous Shear Layer under Dynamic Conditions,” invited contribution, 9^{th} World Congress on Computational Mechanics (WCCM 9), Sydney, Australia, July 19-23 2010.

19. “A Biological Growth Model for Soft Tissue Under Stress Driven and Biological Availability,” invited contribution, 9^{th} World Congress on Computational Mechanics (WCCM 9), Sydney, Australia, July 19-23 2010.

20. “Progress in the Formulation of Finite Elements with Embedded Discontinuities to Model Failure in Solids,” invited contribution, 10^{th} International Conference on Computational Structures Technology (CST2010), ed by B.H.V. Topping et al, Valencia, Spain, September 14-17, 2010.

21. “Elementos Finitos con Discontinuidades Fuertes para el Análisi del Fallo Material en Sólidos,” invited seminar, Instituto Tecnológico and Universidad de Buenos Aires, Buenos Aires, Argentina, November 11 2010.

22. “Finite Elements with Embedded Strong Discontinuities for the Modeling of Failure in Solids,” invited seminar, Department of Civil and Environmental Engineering, Duke University, April 29 2011.

23. “Finite Element Methods in Nonlinear Solid and Structural Dynamics,” invited lecture, International Center of Numerical Methods in Engineering (CIMNE), Polytechnical University of Barcelona, Spain, May 16 2011.

24. “Finite Elements with Embedded Strong Discontinuities for Computational Fracture,” plenary lecture, International Conference on Computational Modeling of Fracture and Failure (CFRAC 2011), Barcelona, Spain, June 6-8 2011.

25. “Finite Elements with Embedded Strong Discontinuities in Computational Failure Mechanics,” plenary lecture, VIII Colombian Congress on Numerical Methods (8CCMN-2011), Medellin, Colombia, August 10-12 2011.

26. “Finite Element Analysis of Failure in Solids and Structures,” plenary lecture, XI International Conference on Computational Plasticity (COMPLAS XI), Barcelona, Spain, September 7-9 2011.

5. Honors and Awards

The PI, Francisco Armero was elected Fellow of the US Association of Computational Mechanics (USACM) in 2011. The Fellows Award was given to him during the 8^{th} US Congress on Computational Mechanics that took place in Minneapolis, July 25, 2011. Additionally, Francisco Armero was awarded in the past the Fellows Award of the International Association for Computational Mechanics (IACM) in July 2004, the Young Investigator Award by the International Association for Computational Mechanics (IACM) in July 2002, the Juan C. Simó award and medal by SEMNI (Spanish Society for Numerical Methods in Engineering) in June 1999, the NSF CAREER award (National Science Foundation) in June 1997, and the ONR Young Investigator Award (Office of the Naval Research) in June 1996. He also received the best paper award for "the most outstanding paper published in *Engineering Computations* in the year 1997".

Additional honors to the PI, Francisco Armero, related to this research project during the performance of the grant include service as:

1. Member of the editorial/advisory board of:
   - University Press of Ecole Centrale de Nantes, January 2011-present.
   - Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería, October 2009-present.
   - Annals of Solid and Structural Mechanics, October 2008-present.
   - Informes de la Construcción, December 2006-present.
   - Computers & Concrete, April 2003-present.
   - Finite Elements in Analysis and Design, February 2002-present.
   - Computer Methods in Applied Mechanics and Engineering, June 2001-present.
   - International Journal for Numerical Methods in Engineering, January 2001-present.
   - Computers & Structures, November 1998-present.
   - International Journal of Numerical Methods in Fluids, November 1997-January 2008.
   - ASCE Journal of Engineering Mechanics, Associate Editor, Sept. 2003-Oct. 2005.
   - Communications in Numerical Methods in Engineering, April 2005-January 2010.
2. Committee service:
   - Committee on Computational Mechanics, ASCE Engineering Mechanics Division (August 1999-present), vice-chair (2003-2004, 2006-2009), chair (August 2004-2006).
   - International Scientific Committee, 10th World Congress on Computational Mechanics (WCCM 10), Sao Paulo, Brazil, July 9-13, 2012.
   - International Scientific Committee of Computational Technologies in Concrete Structures (CTCS'11), Seoul Korea, 18-23 September 2011.

- Technical Advisory Committee, 11^{th} International Conference on Computational Plasticity (COMPLAS XI), Barcelona, Spain, September 7-9, 2011.
- International Scientific Committee, 11th US National Congress on Computational Mechanics (USNCCM 11), Minneapolis MN, July 25-28, 2011.
- Technical Advisory Committee, International Conference on Coupled Problems, Kos Island, Greece, June 20-22, 2011.
- International Scientific Committee, International Conference on Computational Modeling of Fracture and Failure of Materials and Structures (CFRAC 2011), Barcelona, Spain, June 6-8, 2011.
- International Advisory Board, 3^{rd} International Conference on Computational Dynamics and Earthquake Engineering (COMPDYN11), Corfu, Greece, May 26-28, 2011.
- Scientific Advisory Committee, 10^{th} International Conference on Computational Structures Technology (CST2010), Valencia, Spain, September 14-17, 2010.
- Technical Advisory Committee, 10^{th} International Conference on Computational Plasticity (COMPLAS X), Barcelona, Spain, September 2-4, 2009.
- International Advisory Board, Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN09), Island of Rhodes, Greece, June 22-24, 2009.
- Technical Advisory Committee, International Conference on Coupled Problems, Ischia Island, Italy, May 8-10, 2009.
- International Advisory Board, 1^{st} International Conference on Computational Technologies in Concrete Structures (CTCS09), Seoul, Korea, May 2009.
- Scientific Advisory Committee, 9^{th} International Conference on Computational Structures Technology, Athens, Greece, September 2-5, 2008.

3. Organized symposia:

- "Modeling of Damage Evolution and Propagating Discontinuities at Failure," 4 sessions, 21 contributions, XI International Conference on Computational Plasticity (COMPLAS XI), Barcelona, Spain, September 7-9, 2011.
- "Dynamic Fracture," 2 sessions, 11 contributions, International Conference on Computational Modeling of Fracture and Failure of Materials and Structures (CFRAC 2011), Barcelona, Spain, June 6-8, 2011 (total of 11 contributions).
- "Numerical Techniques for the Modeling of Material Failure in Solids," 2 sessions, 11 contributions, 9^{th} World Congress on Computational Mechanics (IX WCCM), Sydney, Australia, July 19-July 23, 2010.
- "Numerical Techniques for the Modeling of Material Failure," 7 sessions, 38 contributions, 8^{th} World Congress on Computational Mechanics (VIII WCCM), Venice, Italy, June 30-July 4, 2008.

6. Personnel

The P.I., Francisco Armero, was supported during the summer months of the project. The graduate student J. Kim has been funded under this grant, and so was partially the Postdoctoral researcher J.S. Valverde Garcia.

7. Acknowledgment/Disclaimer

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